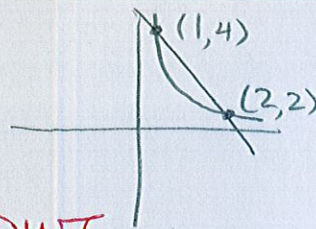


Find the center of mass of the region between the curves $y = \frac{4}{x}$ and $y = 6 - 2x$.

SCORE: ____ / 14 PTS

$$\frac{4}{x} = 6 - 2x$$

$$4 = 6x - 2x^2$$



$$2x^2 - 6x + 4 = 0$$

$$2(x-1)(x-2) = 0$$

$$x = 1, 2$$

ALL ITEMS ① POINT

UNLESS OTHERWISE NOTED

$$A = \int_1^2 (6 - 2x - \frac{4}{x}) dx = (6x - x^2 - 4 \ln|x|) \Big|_1^2 = (12 - 4 - 4 \ln 2) - (6 - 1)$$

$$= 3 - 4 \ln 2$$

$$\int_1^2 x (6 - 2x - \frac{4}{x}) dx = \int_1^2 (6x - 2x^2 - 4) dx = (3x^2 - \frac{2}{3}x^3 - 4x) \Big|_1^2$$

$$= (12 - \frac{16}{3} - 8) - (3 - \frac{2}{3} - 4)$$

$$= \frac{1}{3}$$

$$\frac{1}{2} \int_1^2 ((6-2x)^2 - (\frac{4}{x})^2) dx = \frac{1}{2} \int_1^2 (36 - 24x + 4x^2 - \frac{16}{x^2}) dx$$

$$= \int_1^2 (18 - 12x + 2x^2 - \frac{8}{x^2}) dx$$

$$= (18x - 6x^2 + \frac{2}{3}x^3 + \frac{8}{x}) \Big|_1^2$$

$$= (36 - 24 + \frac{16}{3} + 4) - (18 - 6 + \frac{2}{3} + 8)$$

$$= \frac{2}{3}$$

$$\text{CENTER OF MASS} = \frac{1}{3 - 4 \ln 2} \left(\frac{1}{3}, \frac{2}{3} \right)$$

$$= \left(\frac{1}{9 - 12 \ln 2}, \frac{2}{9 - 12 \ln 2} \right) \textcircled{2}$$

Find the length of the curve $y = \frac{2+x^6}{8x^2}$ from the point $(1, \frac{3}{8})$ to the point $(2, \frac{33}{16})$.

SCORE: ____ / 6 PTS

$$y = \frac{1}{4}x^{-2} + \frac{1}{8}x^4$$

$$y' = \underline{-\frac{1}{2}x^{-3} + \frac{1}{2}x^3}$$

ALL ITEMS

① POINT

$$\begin{aligned} & \int_1^2 \sqrt{1 + \left(-\frac{1}{2}x^{-3} + \frac{1}{2}x^3\right)^2} dx \\ &= \int_1^2 \sqrt{1 + \frac{1}{4}x^{-6} - \frac{1}{2} + \frac{1}{4}x^6} dx \\ &= \int_1^2 \sqrt{\frac{1}{4}x^{-6} + \frac{1}{2} + \frac{1}{4}x^6} dx \\ &= \int_1^2 \left(\frac{1}{2}x^{-3} + \frac{1}{2}x^3\right) dx \\ &= \left(-\frac{1}{4}x^{-2} + \frac{1}{8}x^4\right) \Big|_1^2 \end{aligned}$$

$$= \left(-\frac{1}{16} + 2\right) - \left(-\frac{1}{4} + \frac{1}{8}\right)$$

$$= \frac{31}{16} + \frac{1}{8}$$

$$= \underline{\frac{33}{16}}$$

It takes a patient at most 30 minutes to respond to an injection of a certain drug. A randomly selected patient is given an injection of the drug, and X is the random variable representing the time it takes for patient to respond (measured in units of hours). **SCORE: ___ / 10 PTS**

If the probability density function is given by $f(x) = \begin{cases} \frac{k}{\sqrt{1-x^2}}, & x \in [0, \frac{1}{2}] \\ 0, & x \notin [0, \frac{1}{2}] \end{cases}$ for some constant k , find the mean (average) response time.

$$\int_0^{\frac{1}{2}} \frac{k}{\sqrt{1-x^2}} dx = 1$$

$$k \sin^{-1} x \Big|_0^{\frac{1}{2}} = 1$$

$$k \cdot \frac{\pi}{6} = 1$$

$$k = \frac{6}{\pi}$$

$$\frac{6}{\pi} \int_0^{\frac{1}{2}} \frac{x}{\sqrt{1-x^2}} dx = \frac{6}{\pi} \cdot \frac{-1}{2} \int_1^{\frac{3}{4}} u^{-\frac{1}{2}} du = -\frac{3}{\pi} (2u^{\frac{1}{2}}) \Big|_1^{\frac{3}{4}}$$

$$u = 1 - x^2 \quad \begin{matrix} x = \frac{1}{2} \rightarrow u = \frac{3}{4} \\ x = 0 \rightarrow u = 1 \end{matrix}$$

$$du = -2x dx \quad \text{--- (2) ---}$$

$$-\frac{1}{2} du = x dx$$

$$= -\frac{6}{\pi} \left(\frac{\sqrt{3}}{2} - 1 \right)$$

$$= \frac{6}{\pi} \left(1 - \frac{\sqrt{3}}{2} \right)$$

Hours

ALL ITEMS ① POINT

UNLESS OTHERWISE NOTED